Materials Science Maths for Vacation

Pre-first year

A. Differentiation of elementary functions, product rule, chain rule, physical meaning of differentiation in mechanics

Differentiate the following functions of x with respect to x

1. x^2

2. $\sqrt{3+x}$

3. $\ln(\cos x)$

4. $\frac{e^x}{1+x}$

5. $\sqrt{\frac{1+x}{1-x}}$

6. $\ln(\sec x + \tan x)$

7. The position of an object at time *t* is equal to $3t^3 - 9t^2$. Calculate the times at which (a) the speed of the object is zero, (b) the acceleration of the object is zero.

B. Integration of elementary functions, change of variable, integration by parts, physical meaning of integration in mechanics

8. $\int x^2 \, dx$

9. $\int xe^{-2x} \, dx$

10. $\int \sin x \cos^3 x \, dx$

11. $\int_{-1}^{0} \frac{x}{1+x^2} \, dx$

12. $\int_{-\pi}^{\pi} \sin^2 x dx$

13. $\int \frac{dx}{\sqrt{a^2 - x^2}}$

14. If the velocity of a particle at time *t* is $t^2 + e^t$, how far does it travel between *t*=2 and *t*=4?

C. Properties of functions. Sketching functions. Odd vs even functions. Periodic functions.

Sketch the following functions of *x*, showing where they cross the axes and where they diverge, and state whether they are (a) even, (b) odd and/or (c) periodic.

15. $y = x^3 - x$

16. $y = \frac{1}{\sqrt{1-x^2}}$

17. $y = \frac{\sin x}{x}$

18. $y = \sin(x^2)$

19. $y = \frac{e^x - e^{-x}}{2}$

D. Binomial series. Taylor expansions of elementary function.

20. Expand $(a+b)^6$.

21. What are the first 3 non-vanishing terms in the expansion of $\sqrt{1+x}$ about *x*=1? If one uses the series expansion of $\sqrt{1+x}$ about *x*=0 to evaluate $\sqrt{1.012}$, how many terms are necessary to obtain a value that is correct to 6 decimal places.

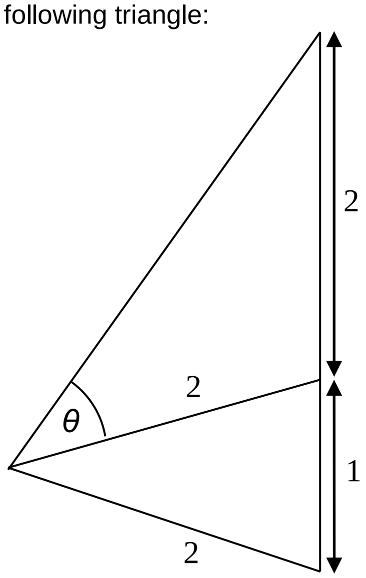
22. What are the first 2 non-vanishing terms in the expansions of $\sin 3x$ and $\cos 3x$, and the first 4 non-vanishing terms in the expansion of e^{3x}?

E. Trigonometric functions. Geometrical meaning of trigonometric functions. Manipulating trigonometric identities.

Given that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and $\cos(A+B) = \cos A \cos B - \sin A \sin B$

23. Prove that $\cos x = \dfrac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$

24. Calculate the angle *θ* in the following triangle:



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F. Rationalising partial fractions.

25. Simplify $\dfrac{1}{x-1} + \dfrac{2}{3-x}$. Hence evaluate $\displaystyle\int \frac{x+1}{(x-1)(3-x)}\,dx$.

G. Understanding and manipulating inequalities.

Find the range of values of *x* which satisfy the following inequalities:

26. $\dfrac{1}{1-x} < 1$

27. $-1 < \dfrac{3x+4}{x-6} < 1$

H. Complex numbers

In this section, the symbol *i* denotes $\sqrt{-1}$.

28. Simplify $(7+i)/(2+3i)$ in the form $a+ib$ and represent your answer on an Argand diagram.

29. Use De Moivre's theorem, $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$, to expand $\cos(4\theta)$ in powers of $\cos\theta$.

30. Find the general solutions for *k* at which $e^{ika} = 1$, $e^{ika} = -1$, $e^{ika} = i$, $e^{ika} = -i$, where *a* is a real constant.

H. Vectors and matrices

In the following, **i**, **j** and **k** are unit vectors along the Cartesian *x*, *y* and *z* axes.

31. What is the position vector of the point $3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ relative to the point $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

32. What is the length of the vector $5\mathbf{i} - \mathbf{j}$ projected in the direction of $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$? Calculate the angle between these two vectors.

33. Find the vector equation of the plane passing through the points (1,0,0), (0,1,0) and (0,0,1) of the Cartesian coordinate system. What is the smallest distance between a point on the plane and the origin?

34. Evaluate $\begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 9 & -6 \\ 2 & 0 \end{pmatrix}$. What is the determinant of the resulting matrix?

35. An operator, represented by the matrix $\begin{pmatrix} 4/5 & 3/5 \\ -3/5 & 4/5 \end{pmatrix}$ acts on the vector $4\mathbf{i} + 3\mathbf{j}$. What is the resulting vector? What operation does the matrix represent? Hence find the inverse of the matrix.

36. Use matrices to solve the following simultaneous equations: $2x + 3y = 5$ and $x - 4y = 9$.