

Units and Some Basics

SI Units

The International System of Units, the SI system¹, is the modern metric system of measurement, established in 1960 by international treaty at the 11th General Conference on Weights and Measures (CGPM²). There are seven SI base units for the following quantities, which are taken to be mutually independent:

Table 1: SI base quantities and their units

Base Quantity	Base Unit	
	Name	Symbol
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s
Thermodynamic Temperature	Kelvin	K
Electric Current	Ampere	A
Luminous Intensity	Candela	cd
Amount of Substance	Mole	mol

The standards which determine the physical magnitude of these base quantities are also decided by international agreement through the CGPM. For example:

- 1 kg is defined to be the mass of the ‘international prototype’, a lump of platinum-iridium alloy kept at the International Bureau of Weights and Measures (BIPM³) at Sèvres, just outside Paris.
- 1 m is the length of the path travelled by light in a vacuum in $1/299\,792\,458$ of a second (note that this means that the speed of light, c , is *fixed by definition* at $299\,792\,458\text{ m s}^{-1}$).
- 1 s is the duration of $9\,192\,631\,770$ periods of the radiation corresponding to the transition between the two hyperfine energy levels of the ground state of the Caesium-133 atom at 0 K.
- 1 K is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of water.
- 1 mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12. This number is *Avogadro’s number*, about 6.022×10^{23} . The elementary entities must be specified (for example, the amount of water molecules in a litre of liquid water is about 55.6 mol, but the amount of hydrogen atoms in the same volume is twice that, 111.2 mol).

Other quantities are derived from the seven base quantities, and many are given their own names and symbols. Some important examples are listed in Table 2.

¹SI is the abbreviation for Le Système International d’Unités

²Conférence Générale des Poids et Mesures

³Bureau International des Poids et Mesures

Table 2: Derived SI quantities and their units

Derived Quantity	Derived Unit		
	Name	Symbol	Base Units
Force	Newton	N	m kg s^{-2}
Pressure	Pascal	Pa	$\text{N m}^{-2} = \text{m}^{-1} \text{kg s}^{-2}$
Energy	Joule	J	$\text{N m} = \text{m}^2 \text{kg s}^{-2}$
Power	Watt	W	$\text{J s}^{-1} = \text{m}^2 \text{kg s}^{-3}$
Frequency	Hertz	Hz	s^{-1}
Electric Charge	Coulomb	C	s A
Electric Potential Difference	Volt	V	$\text{W A}^{-1} = \text{m}^2 \text{kg s}^{-3} \text{A}^{-1}$

SI Prefixes

The series of prefixes and symbols of prefixes to form the names and symbols of decimal multiples and sub-multiples (*i.e.* powers of 10) of SI units is given in Table 3. Some things to note are:

- Prefixes are never mixed within the SI system. For example, the length of the Cl-Cl bond in Cl_2 is 2×10^{-10} m, which can be written as 200 pm (‘200 picometres’) or 0.2 nm (‘0.2 nanometres’) but not 0.2 m μ m (‘0.2 millimicrons’). Similarly, the mass of the moon is about 7.35×10^{22} kg \equiv 73.5 Yg (‘73.5 yottagrams’), not 73.5 Zkg. This is the case even though it is the kilogram which is the base SI unit and not the gram.
- A good way of recalling the base units that comprise a derived SI unit is to relate them through a famous and easy to remember formula. For example,

$$F = ma \rightarrow 1 \text{ N} \equiv 1 (\text{kg})(\text{m s}^{-2}),$$

$$\text{KE} = \frac{1}{2}mv^2 \rightarrow 1 \text{ J} \equiv 1 (\text{kg})(\text{m s}^{-1})^2 \equiv 1 \text{ kg m}^2 \text{ s}^{-2}.$$

- A common source of error in calculations arises from a failure to correctly account for units raised to powers, and care needs to be taken in making conversions between units. For example, $1 \text{ cm}^3 \equiv (10^{-2} \text{ m})^3 \equiv 10^{-6} \text{ m}^3$.

Examples

1. A typical chemical bond vibrates with a period of about 2×10^{-14} s \equiv 20 fs (‘20 femtoseconds’), a frequency of 5×10^{13} s $^{-1}$ \equiv 50 THz (‘50 terahertz’).
2. These prefixes are often used in front of non-SI units as well. For example, the human genome consists of 3164.7 million nucleotide bases (‘A’, ‘C’, ‘G’, ‘T’), which is about 3 ‘gigabases’.
3. Atmospheric pressure at sea level is approximately 100 kPa \equiv 10^5 Pa \equiv 10^5 N m $^{-2}$ \equiv 10 N cm $^{-2}$.

Table 3: SI prefixes (common prefixes in **bold**)

Factor	Prefix	Symbol
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10^1	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

Non-SI units

There are several other unit systems in common use, and conversions between them are given below. Closely related to the SI system is the *cgs system*, another metric system with base units including the centimetre, gram, and second⁴. For example, the derived unit of force in cgs is the dyne: $1 \text{ dyn} \equiv 1 \text{ cm g s}^{-2} \equiv 10^{-5} \text{ N}$.

Americans are inexplicably fond of the *Imperial system*, which used to contain all sorts of silly definitions (an inch was once defined as the width of a man's thumb), but now has a fixed relationship to the SI system (e.g. $1 \text{ in} \equiv 0.0254 \text{ m}$). The shortcomings of the Imperial system are too numerous to go into here, but suffice it to mention that pints are different sizes in the UK (578 ml) and US (473 ml), a pound of gold doesn't weigh the same as a pound of coal, and there are at least three different types of calorie. All scientific calculations should be carried out in SI (or at least metric) units.

The only countries not to have officially adopted SI units as their primary system of measurement are Liberia, Burma, and the United States. Two of those countries are brutal dictatorships with a history of spurning international cooperation. I don't know what Liberia's excuse is.

Length

Distances on the scale of atoms and molecules are often reported in *angstroms*:

⁴the SI system is also known as the *MKS system* for metre, kilogram, second

$$1 \text{ \AA} \equiv 10^{-10} \text{ m} \equiv 0.1 \text{ nm}.$$

Volume

Although not strictly an SI unit name, the litre (symbol: L, l) is in common usage:

$$1 \text{ L} \equiv 1 \text{ dm}^3 \equiv 10^{-3} \text{ m}^3.$$

Mass

Although the SI system suggests that 1000 kg should be equal to 1 megagram, the word 'tonne' (sometimes called a 'metric ton') is often used:

$$1 \text{ t} \equiv 10^3 \text{ kg} \equiv 10^6 \text{ g}.$$

At the other end of the scale, *atomic mass units* are often used when referring to the masses of molecules. 1 u is defined as the one-twelfth of the mass of a ^{12}C atom. Because of the definition of the mole, this means that

$$1 \text{ u} \equiv \frac{1}{1000N_{\text{A}}} \text{ kg} = 1.661 \times 10^{-27} \text{ kg},$$

where $N_{\text{A}} = 6.022 \times 10^{23} \text{ mol}^{-1}$ is Avogadro's constant.

The *molar mass* is the mass of one mole of a substance, usually expressed in g mol^{-1} .

Angle

The SI (derived) unit of plane angle is the radian (symbol: rad). It is still common to see degrees ($^{\circ}$), minutes ($'$), and seconds ($''$) in use, however:

$$\begin{aligned} 1^{\circ} &\equiv (\pi/180) \text{ rad}, \\ 1' &\equiv (1/60)^{\circ} \equiv (\pi/10\,800) \text{ rad}, \\ 1'' &\equiv (1/60)' \equiv (\pi/648\,000) \text{ rad}. \end{aligned}$$

For example, Oxford is at a latitude of $51^{\circ}45'30''$ N, corresponding to a plane angle of 0.903353 rad.

Time

Timescales significantly longer than a second are generally not reported in metric units, but in the familiar minutes, hours and days:

$$\begin{aligned} 1 \text{ min} &\equiv 60 \text{ s}, \\ 1 \text{ hr} &\equiv 60 \text{ min} \equiv 3600 \text{ s}, \\ 1 \text{ day} &\equiv 24 \text{ hr} \equiv 86\,400 \text{ s}. \end{aligned}$$

Temperature

For historical reasons, thermodynamic temperatures are often still reported in degrees Celsius ($^{\circ}\text{C}$). The exact conversion is given by the formula:

$$T/^{\circ}\text{C} = T/K - 273.15.$$

To convert a temperature in degrees Fahrenheit to degrees Celsius, use:

$$T/^{\circ}\text{C} = \frac{5}{9}(T/^{\circ}\text{F} - 32).$$

Pressure

The common non-SI units of pressure are the bar (cgs), atmosphere, Torr, and millimetres of mercury:

$$\begin{aligned} 1 \text{ bar} &\equiv 10^5 \text{ Pa} \equiv 10^5 \text{ N m}^{-2}, \\ 1 \text{ atm} &\equiv 101325 \text{ Pa} \equiv 760 \text{ Torr}, \\ 1 \text{ mmHg} &= 133.3224 \text{ Pa}. \end{aligned}$$

1 mmHg is very nearly (but not exactly) equal to 1 Torr.

Energy

The Imperial unit of energy is the *thermochemical calorie*, defined by:

$$\begin{aligned} 1 \text{ cal} &\equiv 4.184 \text{ J}, \\ 1 \text{ Cal} &\equiv 1 \text{ kcal} \equiv 4184 \text{ J}, \end{aligned}$$

though several other definitions exist: for example, the 15°C calorie is defined as the amount of energy required to heat 1 g of water from 14.5°C to 15.5°C at a constant pressure of 1 atm ($1 \text{ cal}_{15} = 4.1855 \pm 0.0005 \text{ J}$) and the *large IUNS⁵ calorie* ($\equiv 4182 \text{ J}$) is popular as a measure of the energy content of food. It is exactly this sort of lunacy that the modern SI system was designed to avoid. For the small energies involved in individual atoms, the electron volt is a popular unit:

$$1 \text{ eV} = 1.60219 \times 10^{-19} \text{ J};$$

in thermodynamic calculations energies are often reported in kilojoules per mole:

$$1 \text{ kJ mol}^{-1} = 1.6605 \times 10^{-21} \text{ J}.$$

Concentration

In liquid chemistry, concentration is most usually measured in terms of either the *molarity* or *molality* of a solution. The molarity, or molar concentration, of a solution is the number of moles of a substance per dm^3 (*i.e.* litre) of *solution* and thus has units of $\text{M} \equiv \text{mol dm}^{-3}$. Note that this is slightly different from

⁵International Union of Nutritional Sciences

the amount of substance per litre of *solvent* since dissolving a substance in a fixed amount of solvent will change its volume.

Molality is defined as the number of moles of a substance per kg of solvent (not solution), and has units of mol kg^{-1} . It is preferred to molarity because the solvent mass is readily measured and is independent of temperature and pressure, unlike the solution volume.

Since 1 dm^3 of water has a mass close to 1 kg at room temperature and 1 atm pressure, the molarity and molality of a dilute aqueous solution are numerically nearly equal.

Another common measure of concentration is the *mole fraction*, defined as the number of moles of a particular substance divided by the total number of moles of all substances present in a system.

Example

1 L of 40% (by volume) vodka contains approximately 32% ethanol by mass, and weighs 0.988 kg. What is the molar concentration and mole fraction of ethanol in the vodka?

Answer: The mass of ethanol present is

$$M(\text{EtOH}) = 0.32 \times 0.988 \text{ kg} = 0.316 \text{ kg}.$$

The molar mass of ethanol is 46 g mol^{-1} , so the number of moles of ethanol is

$$n(\text{EtOH}) = \frac{0.316 \text{ kg}}{0.046 \text{ kg mol}^{-1}} = 6.87 \text{ mol}$$

Since the volume of the solution is 1 dm^3 , the ethanol concentration is 6.87 M. The mass of water present is $0.988 - 0.316 = 0.672 \text{ kg}$, corresponding to

$$n(\text{H}_2\text{O}) = \frac{0.672 \text{ kg}}{0.018 \text{ kg mol}^{-1}} = 37.3 \text{ mol}$$

and so the mole fraction of ethanol is

$$x(\text{EtOH}) = \frac{n(\text{EtOH})}{n(\text{EtOH}) + n(\text{H}_2\text{O})} = \frac{6.87}{6.87 + 37.3} = 0.16$$

Example

The current concentration of carbon dioxide in the atmosphere is about 385 ppmv (parts per million by volume) and the air number density at sea level is about $2.7 \times 10^{25} \text{ m}^{-3}$ - *i.e.* there is this number of air molecules (of all types) per cubic metre. Calculate the number density and the mass density of CO_2 .

Answer: The number concentration of CO_2 is

$$(385 \times 10^{-6}) \times 2.7 \times 10^{25} \text{ m}^{-3} = 1.0 \times 10^{22} \text{ m}^{-3}.$$

This is a molar CO_2 density of

$$\frac{(1.0 \times 10^{22} \text{ m}^{-3})}{N_A} = 1.7 \times 10^{-2} \text{ mol m}^{-3}$$

or, since the molar mass of CO_2 is 44 g mol^{-1} , a mass density of

$$1.7 \times 10^{-2} \text{ mol m}^{-3} \times (44 \text{ g mol}^{-1}) = 0.76 \text{ g m}^{-3}.$$

Quantity Calculus

Quantity calculus is a powerful way of handling calculations involving physical quantities and their units. A physical quantity is considered to be the product of a numerical value (*i.e.* pure number) and a unit:

$$\text{physical quantity} = \text{numerical value} \times \text{unit}$$

In this way, the units in which a physical quantity is measured are included in its specification, and they can be kept track of in calculations by applying the usual rules of algebra. For example, the wavelength of blue light, λ (the physical quantity) is about 450 nm, which may be written:

$$\lambda = 450 \text{ nm, or equivalently: } \lambda/\text{nm} = 450.$$

Now, since the units m and nm are related by $\text{nm} \equiv 10^{-9} \text{ m}$,

$$\lambda = 4.5 \times 10^{-7} \text{ m, or equivalently: } \lambda/\text{m} = 4.5 \times 10^{-7}$$

In equations, only pure numbers can be manipulated (for example, a number cannot be raised to the power of a distance, and it is strictly meaningless to take the natural logarithm of a temperature). Therefore, it is common in writing equations to specify both the physical quantity and its units. For example, as we have seen, to convert a temperature in kelvin, T, to a temperature in degrees celsius, °C:

$$T/^{\circ}\text{C} = T/\text{K} - 273.15.$$

T is a physical quantity, say 250 K; T/K is a pure number, 250. So to apply the equation, subtracting the *numbers* 250 - 273.15 gives the *number* $T/^{\circ}\text{C} = -23.15$, and the *physical quantity* $T = -23.15^{\circ}\text{C}$.

Quantity calculus is useful in tabulating the numerical values of physical quantities and in labelling the axes of graphs. For example:

T/K	$10^3 \text{ K}/T$	$p \times 10^{-6} / \text{Pa}$	$\ln(p/10^6 \text{ Pa})$
150	6.667	0.85	-0.1625
200	5.000	2.64	0.9708
298	3.336	6.97	1.9416

The usual rules of algebra are followed to interpret tables like this. So, for example, the second entry in the second column, $10^3 \text{ K}/T = 5.000$ implies that $1/T = 5 \times 10^{-3} \text{ K}^{-1}$ which is, indeed, $1/(200 \text{ K})$. An equivalent way of writing this column heading would be $(1/T)/10^{-3} \text{ K}^{-1}$. The corresponding entry in the third column, $p \times 10^{-6} / \text{Pa} = 2.64$ implies a pressure of $p = 2.64/10^{-6} \text{ Pa} \equiv 2.64 \times 10^6 \text{ Pa} \equiv 2.64 \text{ MPa}$.

Example

A second-order reaction $\text{A} + \text{B} \rightarrow \text{P}$ obeys the rate law:

$$r = \frac{d[\text{P}]}{dt} = k[\text{A}][\text{B}]$$

where the units of the rate of reaction, r are conc s^{-1} where the derived SI unit of molar concentration is $\text{conc} = \text{mol m}^{-3}$. Therefore, the rate constant, k , must have units of $\text{conc}^{-1} \text{ s}^{-1} = \text{m}^3 \text{ mol}^{-1} \text{ s}^{-1}$.

Checking for Consistent Units

It is a good idea, at least to begin with, to get into the practice of writing the units along with the value of physical quantities when doing numerical work.

Examples

1. Calculate the equilibrium constant for a reaction with $\Delta_r G^\ominus = 33 \text{ kJ mol}^{-1}$, at 298 K.

Answer: The equilibrium constant is related to the Gibbs Free energy change of a reaction through the formula $\Delta_r G^\ominus = -RT \ln K$, so

$$\begin{aligned} K &= \exp\left(-\frac{\Delta_r G^\ominus}{RT}\right) \\ &= \exp\left(-\frac{33 \times 10^3 \text{ J mol}^{-1}}{(8.314 \text{ J K}^{-1} \text{ mol}^{-1})(298 \text{ K})}\right) \\ &= 1.6 \times 10^{-6}. \end{aligned}$$

Note that an equilibrium constant calculated in this way is dimensionless.

2. What is the mean free path (average distance between collisions) of N_2 molecules at $T = 298 \text{ K}$ and $p = 1 \text{ bar}$. The collision cross section of N_2 is $\sigma = 0.43 \text{ nm}^2$.

Answer: The mean free path,

$$\begin{aligned} \lambda &= \frac{k_B T}{\sqrt{2} \sigma p} \\ &= \frac{(1.381 \times 10^{-23} \text{ J K}^{-1})(298 \text{ K})}{\sqrt{2}(0.43 \times 10^{-18} \text{ m}^2)(1 \times 10^5 \text{ Pa})} \\ &= 6.8 \times 10^{-8} \text{ m} = 68 \text{ nm}. \end{aligned}$$

We can be sure that the answer will be in SI units (m) if we have used SI units throughout the calculation, but to check the units convert them to base SI units:

$$\frac{(\text{J K}^{-1})(\text{K})}{(\text{m}^2)(\text{Pa})} = \frac{\text{J}}{(\text{m}^2)(\text{N m}^{-2})} = \frac{\text{J}}{\text{N}} = \frac{\text{kg m}^2 \text{ s}^{-2}}{\text{kg m s}^{-2}} = \text{m}$$

Standard Conditions

There are two important sets of conditions which are referred to as ‘standard’:

- Standard Temperature and Pressure (STP) is defined by IUPAC⁶ to be a temperature of $0^\circ\text{C} \equiv 273.15 \text{ K}$ and a pressure of $1 \text{ bar} \equiv 10^5 \text{ Pa}$. However, the older definition (prior to 1982) of standard pressure as $1 \text{ atm} \equiv 1.01325 \times 10^5 \text{ Pa}$ is still encountered.
- Standard Ambient Temperature and Pressure (SATP) refers to $25^\circ\text{C} \equiv 298.15 \text{ K}$ and $1 \text{ bar} \equiv 10^5 \text{ Pa}$ pressure. 25°C is sometimes referred to as ‘room temperature’ (though that would have to be quite a warm room).

⁶International Union of Pure and Applied Chemistry

The standard pressure of 1 bar is frequently used in thermodynamics, and is given its own symbol:

$$p^\ominus \equiv 1 \text{ bar.}$$

The *standard state* of a substance is its pure form at $p^\ominus \equiv 1 \text{ bar}$ and *at a specified temperature*.

Avogadro's principle states that equal volumes of gases at the same temperature and pressure contain the same number of molecules, regardless of the identity of the gases. It applies to ideal (or perfect) gases, and is a good approximation for real gases in many instances. Stated another way, Avogadro's principle implies that a given amount of any gas at a specified temperature and pressure occupies a particular volume. Specifically one mole of any ideal gas has a molar volume, V_m , equal to:

$$\begin{aligned} V_m &= 22.414 \text{ L mol}^{-1} \text{ at } 273.15 \text{ K, } 1 \text{ atm} \\ V_m^\ominus &= 24.711 \text{ L mol}^{-1} \text{ at STP (273.15 K, } 1 \text{ bar)} \\ V_m^\ominus &= 24.790 \text{ L mol}^{-1} \text{ at SATP (298.15 K, } 1 \text{ bar)} \end{aligned}$$

In the case of liquid solutions, the standard state of a solute may be defined as unit activity in a specified solvent at a given temperature, $a = 1$. For very dilute aqueous solutions, activities are almost equal to molalities, m , in the sense:

$$a \approx m/m^\ominus$$

where the standard molality, $m^\ominus \equiv 1 \text{ mol kg}^{-1}$, and to molarities (amount concentrations), c :

$$a \approx c/c^\ominus$$

where the standard concentration, $c^\ominus \equiv 1 \text{ M} \equiv 1 \text{ mol dm}^{-3}$.

Errors and Uncertainty

All of science is based on empirical observations - that is, measurements made with physical instruments. These are inevitably imperfect in the sense that they provide an estimate of an assumed true value of a physical quantity. For example, the potential difference across a given electrochemical cell has some value which we may attempt to measure with a voltmeter. A number will be displayed on this instrument and we need to be able to state how well we think this represents the true voltage. At this point, it is important to distinguish between accuracy and precision. Suppose the display reads 2.134 V, and that a series of repeated measurements gives the same reading. The *precision* of the measurement may then be stated as $\pm 0.001 \text{ V}$, but there is a further question to ask: how close is this value to the true value - in other words, how *accurate* is the voltmeter? In general this can be quite a difficult question to answer, but an indication is usually given by the manufacturer's specification of the instrument. For example, it may have been calibrated against a more accurate instrument and found to have an accuracy of 0.2 %. In this case, the accuracy of our measurement is about 0.004 V, and the displayed digits (precision) have given a false impression of the uncertainty in our measurement. The true uncertainty (or error) may be indicated along with the measurement by writing $2.134 \pm 0.004 \text{ V}$. Now, assume we have made a series of measurements in an experiment, and determined the error associated with each one. Typically, we will need to perform

some kind of algebraic manipulation of the data (adding it together, multiplying by constants, taking square roots, etc...) in order to determine some physical quantity of interest. The error associated with this final result is determined by a procedure known as *propagation of errors*. For example, if we have two independent (*i.e.* uncorrelated) measurements A and B , with associated errors δA and δB , and add them to form a quantity X , the uncertainty in X , δX is given by: $(\delta X)^2 = (\delta A)^2 + (\delta B)^2$. Some more relations are given in Table 4.

Table 4: Propagation of uncorrelated errors

Algebraic relationship [†]	Error in the result, δX
$X = A \pm B$	$(\delta X)^2 = (\delta A)^2 + (\delta B)^2$
$X = cA$	$(\delta X) = c(\delta A)$
$X = c(A \times B)$ or $X = c(A/B)$	$(\delta X/X)^2 = (\delta A/A)^2 + (\delta B/B)^2$
$X = e^{cA}$	$(\delta X/X) = c\delta A$
$X = \ln(cA)$	$(\delta X) = (\delta A/A)$
$X = A^c$	$(\delta X/X) = c(\delta A/A)$

[†] c is a precisely known constant

A full analysis of a calculation including propagation of errors requires estimates of uncertainties in the input quantities, which are often not given in simple problems. Nevertheless, answers to these problems should be expressed in a way which gives some indication of their likely accuracy. A good rule of thumb is to take any physical constants you need to one or two more significant figures than the data given, and then to round down the final answer to the same number of significant figures as this data.

Example

The air pressure at an altitude z is given approximately by the formula

$$p(z) = p(0)\exp(-z/H),$$

where $p(0) = 1$ atm is the air pressure at sea level, and H is the scale height, estimated to be 7 ± 1.5 km. What is the air pressure at the summit of Mt. Everest (8848 m)?

Answer: Using quantity calculus, the equation for $p(z)$ may be written:

$$p(z) = (1 \text{ atm})\exp[-z/(7 \text{ km})]$$

and so $p(8.848 \text{ km}) = 0.28$ atm.

To assess the uncertainty in this result, note that the greatest source of error is in the estimate of H . The error in the quantity $X = z/H$ is given by

$$(\delta X/X)^2 = (\delta z/z)^2 + (\delta H/H)^2,$$

but since $(\delta z/z)^2$ is negligible (the height of Everest is known to the nearest metre):

$$\delta X = \frac{\delta H}{H} \times X = \frac{1.5}{7} \times \frac{8.848}{7} = 0.271.$$

This error propagates in the calculation of $Y = \exp(-X)$, such that $(\delta Y/Y) = \delta X$ and thus $\delta Y = 0.271(0.28 \text{ atm}) = 0.076$ atm. The final result may therefore be quoted:

$$p = 0.28 \pm 0.08 \text{ atm.}$$

Further Reading

1. The website of the International Bureau of Weights and Measures (BIPM, the Bureau International des Poids et Mesures) is <http://www.bipm.org/en/si/>.
2. *Quantities, Units and Symbols in Physical Chemistry (The Green Book)*, 2nd ed., I. Mills, T. Cvitas, K. Homann, N. Kallay, K. Kuchitsu, Blackwell Science (1993).
3. *The Choice of Names and Symbols for Quantities in Chemistry*, I. M. Mills, *Journal of Chemical Education* **66**, 887-889 (1989).
4. *On the History of Quantity Calculus and the International System*, J. de Boer, *Metrologia* **31**, 405-429 (1995).
5. *The Cambridge Handbook of Physics Formulas*, G. Woan, Cambridge University Press (2000).

Exercises

1. Verify that the following formula for the speed of sound in a gas, c , gives an answer with the correct units:

$$c = \sqrt{\frac{\gamma p}{\rho}},$$

where p is the pressure of the gas, ρ is its density, and γ is the (dimensionless) adiabatic index.

2. A *harmonic oscillator* may be thought of as a spring with the property that it exerts a restoring force, F , proportional to its extension or compression, x , relative to its unstretched length: $F = -kx$, where k is the *spring force constant*.

- (a) What are the units of k ? Show that they consistent with its use in defining the potential energy stored in the spring, $V = \frac{1}{2}kx^2$.
- (b) Show that the following expression for the vibrational frequency, ν , of a mass, m , attached to a harmonic oscillator gives a quantity with the units Hz:

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

3. The mean speed of the molecules of an ideal gas is given by

$$\bar{c} = \sqrt{\frac{8k_{\text{B}}T}{\pi m}},$$

where m is the molecular mass. Use this formula to calculate the mean speed of nitrogen molecules at 298 K. The *molar* mass of N_2 is 28.02 g mol^{-1} .

4. What temperature has the same numerical value in $^{\circ}\text{C}$ and $^{\circ}\text{F}$?

5. Car airbags inflate with nitrogen gas produced by the rapid thermal decomposition of sodium azide, NaN_3 . What volume of gas (at SATP) is produced from the complete decomposition of 120 g of NaN_3 ? Take molar masses $M(\text{Na}) = 23 \text{ g mol}^{-1}$ and $M(\text{N}) = 14 \text{ g mol}^{-1}$, and assume that the nitrogen gas behaves ideally ($pV = nRT$).
6. *Harder.* Why is the sky blue?

Hints: the ratio of the intensity of light scattered by small particles such as air molecules to its incident intensity, I'/I might depend on the following quantities: c , the speed of light; λ , the wavelength of the light; r , the distance of the scattering particles from the observer; V , the volume of each particle; D_s the density of the scattering particles; and D , the density of the surrounding medium.

The intensity of light is proportional to the square of the amplitude of the light wave, so $I'/I \propto (A'/A)^2$, and it is reasonable to expect

$$\frac{A'}{A} \propto V \quad \text{and} \quad \frac{I'}{I} \propto \frac{1}{r^2}$$

Derive an expression for the dependence of the (dimensionless) ratio I'/I on the above quantities by considering the units of each and arriving at a unique expression involving a power of λ .